H

National
Qualifications
2016

2016 Mathematics

Higher Paper 1

Finalised Marking Instructions

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times 6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x = 1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 1 \text{ or } 3$

(k) **Horizontal/vertical marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

$$\begin{array}{ccc} & \bullet^5 & \bullet^6 \\ \bullet^5 & x = 2 & x = -4 \\ \bullet^6 & y = 5 & y = -7 \end{array}$$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$

$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)		•1 interpret recurrence relation and calculate u_4	•1 $u_4 = 12$	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•2 communicate condition for limit to exist	•2 A limit exists as the recurrence relation is linear and $-1 < \frac{1}{3} < 1$	**1**

Notes:

1. On this occasion for •2 accept:

   any of $-1 < \frac{1}{3} < 1$ or $\left|\frac{1}{3}\right| < 1$ or $0 < \frac{1}{3} < 1$ with no further comment;

   or statements such as:

   "$\frac{1}{3}$ lies between -1 and 1" or "$\frac{1}{3}$ is a proper fraction"

2. •2 is not available for: $-1 \leq \frac{1}{3} \leq 1$ or $\frac{1}{3} < 1$

   or statements such as:

   "It is between -1 and 1" or "$\frac{1}{3}$ is a fraction"

3. Candidates who state $-1 < a < 1$ can only gain •2 if it is **explicitly stated** that $a = \frac{1}{3}$.

Commonly Observed Responses:

Candidate A	Candidate B
$a = \frac{1}{3}$ $-1 < a < 1$ so a limit exists. •2 ✓	$u_{n+1} = au_n + b$ $u_{n+1} = \frac{1}{3}u_n + 10$ $-1 < a < 1$ so a limit exists. •2 ^

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		•3 Know how to calculate limit	•3 $\dfrac{10}{1-\frac{1}{3}}$ or $L = \frac{1}{3}L + 10$	**2**
			•4 calculate limit	•4 15	

Notes:

4. Do not accept $L = \dfrac{b}{1-a}$ with no further working for •3.
5. •3 and •4 are not available to candidates who conjecture that $L = 15$ following the calculation of further terms in the sequence.
6. For $L = 15$ with no working, award 0/2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.			•¹ find the centre •² calculate the radius •³ state equation of circle	•¹ $(-3,4)$ stated or implied by •³ •² $\sqrt{17}$ •³ $(x+3)^2+(y-4)^2=17$ or equivalent	**3**

Notes:

1. Accept $\frac{\sqrt{68}}{2}$ for •².
2. •³ is not available to candidates who do not simplify $\left(\sqrt{17}\right)^2$ or $\left(\frac{\sqrt{68}}{2}\right)^2$.
3. •³ is not available to candidates who do not attempt to half the diameter.
4. •³ is not available to candidates who use either A or B for the centre.
5. •³ is not available to candidates who substitute a negative value for the radius.
6. •²&•³ are not available to candidates if the diameter or radius appears ex nihilo.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			•¹ start to integrate •² complete integration	•¹ $\ldots\times\sin(4x+1)$ •² $2\sin(4x+1)+c$	**2**

Notes:

1. An answer which has not been fully simplified, eg $\frac{8\sin(4x+1)}{4}+c$ or $\frac{4\sin(4x+1)}{2}+c$, does not gain •².
2. Where candidates have differentiated throughout, or in part (indicated by the appearance of a negative sign or $\times 4$), see candidates A to F.
3. No marks are available for a line of working containing $\sin(4x+1)^2$ or for any working thereafter.

Commonly Observed Responses:

Candidate A Differentiated throughout: $-32\sin(4x+1)+c$ award 0/2	**Candidate C** Differentiated in part: $32\sin(4x+1)+c$ award 1/2	**Candidate E** Differentiated in part: $-2\sin(4x+1)+c$ award 1/2
Candidate B Differentiated throughout: $-32\sin(4x+1)$ award 0/2	**Candidate D** Differentiated in part: $32\sin(4x+1)$ award 0/2	**Candidate F** Differentiated in part: $-2\sin(4x+1)$ award 0/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)			**Method 1:**	**3**
			$\bullet^1$ equate composite function to x $\bullet^2$ write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$ $\bullet^3$ state inverse function	$\bullet^1$ $f\left(f^{-1}(x)\right) = x$ $\bullet^2$ $3f^{-1}(x)+5 = x$ $\bullet^3$ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 2:	**3**
			$\bullet^1$ write as $y=3x+5$ and start to rearrange $\bullet^2$ complete rearrangement $\bullet^3$ state inverse function	$\bullet^1$ $y-5=3x$ $\bullet^2$ $x=\dfrac{y-5}{3}$ $\bullet^3$ $f^{-1}(x)=\dfrac{x-5}{3}$	
				Method 3	**3**
			$\bullet^1$ interchange variables $\bullet^2$ complete rearrangement $\bullet^3$ state inverse function	$\bullet^1$ $x=3y+5$ $\bullet^2$ $\dfrac{x-5}{3}=y$ $\bullet^3$ $f^{-1}(x)=\dfrac{x-5}{3}$	

Notes:

1. $y=\dfrac{x-5}{3}$ does not gain $\bullet^3$.
2. At $\bullet^3$ stage, accept f^{-1} expressed in terms of any dummy variable eg $f^{-1}(y)=\dfrac{y-5}{3}$.
3. $f^{-1}(x)=\dfrac{x-5}{3}$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$$\times 3 \quad +5$$

$$x \to 3x \to 3x+5 = f(x)$$

$$\div 3 \quad -5$$

$\bullet^1$ ✓

$\dfrac{x-5}{3}$ $\bullet^2$ ✓

$f^{-1}(x)=\dfrac{x-5}{3}$ $\bullet^3$ ✓

$\bullet^1$ awarded for knowing to perform inverse operations in reverse order.

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ correct value	$\bullet^1$ 2	**1**

Notes:

Commonly Observed Responses:

Candidate B

$g(x) = 3x + 1$

$g(2) = 3 \times 2 + 1 = 7$

$g^{-1}(x) = \dfrac{x-1}{3}$

$g^{-1}(7) = \dfrac{7-1}{3} = 2$ $\bullet^4$ ✘

If the candidate had followed this by stating that this would be true for all functions g for which $g(2) = 7$ and g^{-1} exists then $\bullet^4$ would be awarded.

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		$\bullet^1$ identify pathway $\bullet^2$ state $\overrightarrow{FH}$	$\bullet^1$ $\overrightarrow{FG}+\overrightarrow{GH}$ $\bullet^2$ $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$	**2**

Notes:

1. Award $\bullet^1$ for $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})$.
2. For $\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ without working, award both $\bullet^1$ and $\bullet^2$.
3. Accept, throughout the question, solutions written as column vectors.
4. $\bullet^2$ is not available for adding or subtracting vectors within an invalid strategy.
5. Where candidates choose specific points consistent with the given vectors only $\bullet^1$ and $\bullet^4$ are available. However, should the statement 'without loss of generality' precede the selected points then all 4 marks are available.

Commonly Observed Responses:

Candidate A

$\overrightarrow{FH}=\overrightarrow{FG}+\overrightarrow{EH}$ $\quad\bullet^1$ ✗

$\begin{pmatrix}-2\\-6\\3\end{pmatrix}+\begin{pmatrix}2\\3\\1\end{pmatrix}$

$\begin{pmatrix}0\\-3\\4\end{pmatrix}$ $\quad\bullet^2$ $\boxed{\checkmark 2}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ identify pathway $\bullet^2$ $\overrightarrow{FE}$	$\bullet^1$ $\overrightarrow{FH}+\overrightarrow{HE}$ or equivalent $\bullet^2$ $-\mathbf{i}-5\mathbf{k}$	**2**

Notes:

6. Award $\bullet^3$ for $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(\mathbf{i}+3\mathbf{j}-4\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})-(2\mathbf{i}+3\mathbf{j}+\mathbf{k})$
   or $(-2\mathbf{i}-6\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+9\mathbf{j}-7\mathbf{k})+(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})$.
7. For $-\mathbf{i}-5\mathbf{k}$ without working, award 0/2.
8. $\bullet^4$ is not available for simply adding or subtracting vectors. There must be evidence of a valid strategy at $\bullet^3$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			•1 substitute for y	•1 $x^2+(3x-5)^2+2x-4(3x-5)-5...$	**5**
			Method 1 & 2	**Method 1**	
			•2 express in standard quadratic form •3 factorise or use discriminant	•2 $10x^2-40x+40$ •3 $10(x-2)^2$ $\Big\} = 0$	
			•4 interpret result to demonstrate tangency	•4 only one solution implies tangency (or repeated factor implies tangency)	
			•5 find coordinates	•5 $x=2,\ \ y=1$	
				Method 2	
				•2 $10x^2-40x+40=0$ **stated explicitly**	
				•3 $(-40)^2-4\times10\times40$ or $(-4)^2-4\times1\times4$	
				•4 $b^2-4ac=0$ so line is a tangent	
				•5 $x=2,\ \ y=1$	
			Method 3	**Method 3**	
			•1 make inference and state m_{rad}	•1 If $y=3x-5$ is a tangent, $m_{rad}=\frac{-1}{3}$	
			•2 find the centre and the equation of the radius •3 solve simultaneous equations	•2 $(-1,2)$ and $3y=-x+5$ •3 $\begin{matrix}3y=-x+5\\y=3x-5\end{matrix}\rightarrow(2,1)$	
			•4 verify location of point of intersection	•4 check $(2,1)$ lies on the circle.	
			•5 communicates result	•5 $\therefore$ the line is a tangent to the circle	

Notes:

1. In Method 1 $"=0"$ must appear at •2 or •3 stage for •2 to be awarded.
2. Award •**3** and •**4** for correct use of quadratic formula to get equal (repeated) roots $\Rightarrow$ line is a tangent.

Question	Generic Scheme	Illustrative Scheme	Max Mark

Commonly Observed Responses:

Candidate A

$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •[1] ✓

$10x^2-40x+40=0$ •[2] ✓

$b^2-4ac=(-40)^2-4\times10\times40=0 \Rightarrow$ tgt •[3] ✓

Candidate B

$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •[1] ✓

$10x^2-40x+40$ •[2] ^

$b^2-4ac=(-40)^2-4\times10\times40=0 \Rightarrow$ tgt •[3] $\boxed{\checkmark 1}$

Candidate C

$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •[1] ✓

$x^2+\underline{9x^2+25}+2x-12x+20-5=0$

$10x^2-10x+40=0$ •[2] ✗

$b^2-4ac=(-10)^2-4\times10\times40=-1500 \Rightarrow$

no real roots so line is not a tangent •[3] $\boxed{\checkmark 1}$

•[4] and •[5] are unavailable.

Candidate D

$x^2+(3x-5)^2+2x-4(3x-5)-5=0$ •[1] ✓

$10x^2-40x+40=0$ •[2] ✓

$10(x-2)^2$ •[3] ✓

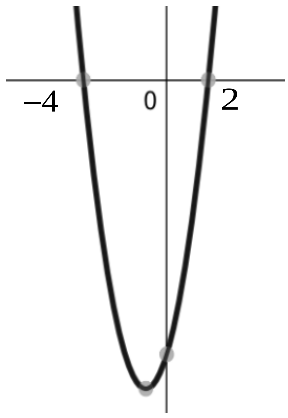
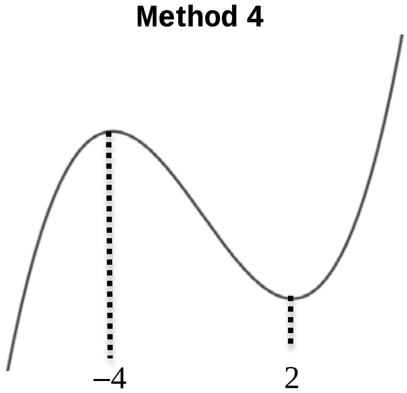
Repeated root $\Rightarrow$ Only one point of contact. •[4] ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
9	(a)		•[1] know to and differentiate one term	•[1] eg $f'(x)=3x^2...$	4
			•[2] complete differentiation and equate to zero	•[2] $3x^2+6x-24=0$	
			•[3] factorise derivative	•[3] $3(x+4)(x-2)$	
			•[4] process for x	•[4] -4 and 2	

Notes:

1. •[2] is only available if $"=0"$ appears at •[2] or •[3] stage.
2. •[3] is available for substituting correctly in the quadratic formula.
3. At •[3] do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 3.
4. •[3] and •[4] are not available to candidates who arrive at a linear expression at •[2].

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
(b)	●[5] know how to identify where curve is increasing	●[5] **Method 1**  **Method 2** $3x^2+6x-24>0$ **Method 3** Table of signs for a derivative – see the additional page for acceptable responses. **Method 4** 	2
	●[6] state range	●[6] $x<-4$ and $x>2$	

Notes:

5. For $x<-4$ and $x>2$ without working award 0/2.
6. $2<x<-4$ does not gain ●[6].

Commonly Observed Responses:

Table of signs for a derivative - acceptable responses.

x	-4^-	-4	-4^+
$\frac{dy}{dx}$ or $f'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	2^-	2	2^+
$\frac{dy}{dx}$ or $f'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

x	$\rightarrow$	-4	$\rightarrow$
$\frac{dy}{dx}$ or $f'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$ or $f'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Arrows are taken to mean "in the neighbourhood of"

x	a	-4	b	2	c
$\frac{dy}{dx}$ or $f'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Where: $a < -4$, $-4 < b < 2$, $c > 2$

Since the function is continuous '$-4 < b < 2$' is acceptable.

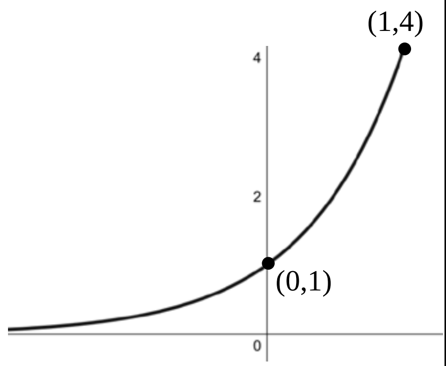
x	$\rightarrow$	-4	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$ or $f'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is continuous '$-4 \rightarrow 2$' is acceptable.

General Comments

- Since this question refers to both y and $f(x)$, $\frac{dy}{dx}$ and $f'(x)$ are accepted.
- The row labelled 'shape' or 'slope' is not required in this question since the sign of the derivative is sufficient to indicate where the function is increasing.
- For this question, do not penalise the omission of 'x' on the top row of the table.

<table>
<tr><th>Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td>10.</td><td>●¹ graph reflected in $y = x$

●² correct annotation</td><td>●¹

●² $(0,1)$ and $(1,4)$</td><td>2</td></tr>
</table>

Notes:

1. For ●¹ accept any graph of the correct shape and orientation which crosses the $y-\text{axis}$. The graph must not cross the $x-\text{axis}$.
2. Both $(0,1)$ and $(1,4)$ must be marked and labelled on the graph for ●² to be awarded.
3. ●² is only available where the candidate has attempted to reflect the given curve in the line $y = x$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		●[1] interpret ratio ●[2] determine coordinates	●[1] $\frac{1}{3}$ ●[2] $(2,1,0)$	**2**

Notes:

1. ●[1] may be implied by ●[2] or be evidenced by their working.
2. For $(3,-1,2)$ award 1/2.
3. For $(2,1,0)$ without working award 2/2.
4. $\begin{pmatrix}2\\1\\0\end{pmatrix}$ gains 1/2.
5. $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ gains 0/2.

Commonly Observed Responses:

Candidate A	**Candidate B**
$\overrightarrow{BC}=\frac{1}{3}\overrightarrow{AC}$ ●[1] ✘ $(3,-1,2)$ ●[2] $\boxed{✓1}$	$\frac{\overrightarrow{AB}}{\overrightarrow{BC}}=\frac{1}{2}$ $2\overrightarrow{AB}=\overrightarrow{BC}$ $2(\mathbf{b}-\mathbf{a})=\mathbf{c}-\mathbf{b}$ $3\mathbf{b}=\mathbf{c}+2\mathbf{a}$ ●[1] ✓ $3\mathbf{b}=\begin{pmatrix}6\\3\\0\end{pmatrix}$ $\mathbf{b}=\begin{pmatrix}2\\1\\0\end{pmatrix}$ $B(2,1,0)$ ●[2] ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[1] find $\overrightarrow{AC}$	•[1] $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$	**3**
			•[2] find $\lvert\overrightarrow{AC}\rvert$	•[2] 9	
			•[3] determine k	•[3] $\frac{1}{9}$	

Notes:

6. Evidence for •[3] may appear in part (a).
7. •[3] may be implied at •[4] stage by :
   - $\sqrt{3^2 + (-6)^2 + 6^2}$
   - $\sqrt{3^2 - 6^2 + 6^2} = 9$
   - $\sqrt{3^2 + -6^2 + 6^2} = 9$.
8. $\sqrt{81}$ must be simplified at the •[4] or •[5] stage for •[4] to be awarded.
9. •[5] can only be awarded as a consequence of a valid strategy at •[4]. k must be > 0 .

Commonly Observed Responses:

Candidate A	**Candidate B**	**Candidate C**
$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$ •[4] ✓	$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$ •[4] ✓2	$\lvert\overrightarrow{AC}\rvert = \sqrt{81}$ •[4] ✓2
$\frac{1}{9}$ •[5] ✓	$\frac{1}{\sqrt{81}}$ •[5] ✓1	•[5] ^

ALTERNATIVE STRATEGY

Where candidates use the distance formulae to determine the distance from A to C, award •[3] for $AC = \sqrt{3^2 + 6^2 + 6^2}$.

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		•1 interpret notation	•1 $2(3-x)^2-4(3-x)+5$	**2**
			•2 demonstrate result	•2 $18-12x+2x^2-12+4x+5$ leading to $2x^2-8x+11$	

Notes:

1. At •2 there must be a relevant intermediate step between •1 and the final answer for •2 to be awarded.
2. $f(3-x)$ alone is not sufficient to gain •1.
3. Beware of candidates who fudge their working between •1 and •2.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)			**Method 1**	**3**
			•1 identify common factor	•1 $2\left[x^2-4x\ldots\right.$ stated or implied by •2	
			•2 start to complete the square	•2 $2(x-2)^2\ldots$	
			•3 write in required form	•3 $2(x-2)^2+3$	
				Method 2	
			•1 expand completed square form	•1 $px^2+2pqx+pq^2+r$	
			•2 equate coefficients	•2 $p=2,\ 2pq=-8,\ pq^2+r=11$	
			•3 process for q and r and write in required form	•3 $2(x-2)^2+3$	

Notes:

4. At •5 $2(x+(-2))^2+3$ must be simplified to $2(x-2)^2+3$.
5. $2(x-2)^2+3$ with no working gains •5 only; however, see Candidate G.
6. Where a candidate has used the function they arrived at in part (a) as $h(x)$, •3 is not available. However, •4 and •5 can still be gained for dealing with an expression of equivalent difficulty.
7. •5 is only available for a calculation involving both the multiplication and addition of integers.

Question	Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:			

Candidate A $2\left(x^2-4x+\frac{11}{2}\right)$ ●³ ✓ $2\left(x^2-4x+4-4+\frac{11}{2}\right)$ ●⁴ not awarded at this line. $2(x-2)^2+\frac{3}{2}$ ●⁴ ✓ ●⁵ ✗	**Candidate B** $2x^2-8x+11=2(x-4)^2-16+11$ ●³ ✗ ●⁴ ✗ $=2(x-4)^2-5$ ●⁵ $\boxed{✓2}$
Candidate C $px^2+2pqx+pq^2+r$ ●³ ✓ $p=2,\ 2pq=-8,\ q^2+r=11$ ●⁴ ✗ $p=2,\ q=-2,\ r=7$ $2(x-2)^2+7$ ●⁵ $\boxed{✓1}$	**Candidate D** $2\left[\left(x^2-8x\right)+11\right]$ ●³ ✗ $2\left[(x-4)^2-16\right]+11$ ●⁴ $\boxed{✓1}$ $2(x-4)^2-21$ ●⁵ $\boxed{✓1}$
Candidate E $p(x+q)^2+r=px^2+2pqx+pq^2+r$ ●³ ✓ $p=2,\ 2pq=-8,\ pq^2+r=11$ ●⁴ ✓ $q=-2,\ r=3$ ●⁵ ✓ ●⁵ is awarded as all working relates to completed square form	**Candidate F** $px^2+2pqx+pq^2+r$ ●³ ✓ $p=2,\ 2pq=-8,\ pq^2+r=11$ ●⁴ ✓ $q=-2,\ r=3$ ●⁵ ✗ ●⁵ is lost as no reference is made to completed square form
Candidate G $2(x-2)^2+3$ Check: $2\left(x^2-4x+4\right)+3$ $=2x^2-8x+8+3$ $2x^2-8x+11$ Award 3/3	**Candidate H** $2x^2-8x+11$ $=2(x-2)^2-4+11$ ●³ ✓ ●⁴ ✓ $=2(x-2)^2+7$ ●⁵ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.			$\bullet^1$ calculate lengths AC and AD	$\bullet^1$ $\text{AC} = \sqrt{17}$ and $\text{AD} = 5$ stated or implied by $\bullet^3$	**5**
			$\bullet^2$ select appropriate formula and express in terms of p and q	$\bullet^2$ $\cos q \cos p + \sin q \sin p$ stated or implied by $\bullet^4$	
			$\bullet^3$ calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$	$\bullet^3$ $\cos p = \dfrac{4}{\sqrt{17}}$, $\cos q = \dfrac{4}{5}$ $\sin p = \dfrac{1}{\sqrt{17}}$, $\sin q = \dfrac{3}{5}$	
			$\bullet^4$ calculate other two and substitute into formula	$\bullet^4$ $\dfrac{4}{5}\times\dfrac{4}{\sqrt{17}}+\dfrac{3}{5}\times\dfrac{1}{\sqrt{17}}$	
			$\bullet^5$ arrange into required form	$\bullet^5$ $\dfrac{19}{5\sqrt{17}}\times\dfrac{\sqrt{17}}{\sqrt{17}}=\dfrac{19\sqrt{17}}{85}$ or $\dfrac{19}{5\sqrt{17}}=\dfrac{19\sqrt{17}}{5\times17}=\dfrac{19\sqrt{17}}{85}$	

Notes:

1. For any attempt to use $\cos(q-p)=\cos q-\cos p$, only $\bullet^1$ and $\bullet^3$ are available.
2. At the $\bullet^3$ and $\bullet^4$ stages, do not penalise the use of fractions greater than 1 resulting from an error at $\bullet^1$. $\bullet^5$ will be lost.
3. Candidates who write $\cos\left(\dfrac{4}{5}\right)\times\cos\left(\dfrac{4}{\sqrt{17}}\right)+\sin\left(\dfrac{3}{5}\right)\times\sin\left(\dfrac{1}{\sqrt{17}}\right)$ gain $\bullet^1$, $\bullet^2$ and $\bullet^3$.

   $\bullet^4$ and $\bullet^5$ are unavailable.
4. Clear evidence of multiplying by $\dfrac{\sqrt{17}}{\sqrt{17}}$ must be seen between $\bullet^4$ and $\bullet^5$ for $\bullet^5$ to be awarded.
5. $\bullet^4$ implies $\bullet^1$, $\bullet^2$ and $\bullet^3$.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$\dfrac{4}{5}\times\dfrac{4}{\sqrt{17}}+\dfrac{3}{5}\times\dfrac{1}{\sqrt{17}}$	$\bullet^4$ ✓	$\text{AC}=\sqrt{17}$ and $\text{AD}=\sqrt{21}$	$\bullet^1$ ✗
$\dfrac{19}{5\sqrt{17}}\times\sqrt{17}$		$\cos q\cos p+\sin q\sin p$	$\bullet^2$ ✓
$\dfrac{19\sqrt{17}}{85}$	$\bullet^5$ ✗	$\cos p=\dfrac{4}{\sqrt{17}}$ $\sin p=\dfrac{1}{\sqrt{17}}$	$\bullet^3$ ✓
		$\dfrac{\sqrt{17}}{\sqrt{21}}\times\dfrac{4}{\sqrt{17}}+\dfrac{2}{\sqrt{21}}\times\dfrac{1}{\sqrt{17}}$	$\bullet^4$ ✗
		$=\ldots\bullet^5$ not available	

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		•¹ state value	•¹ 2	**1**

Notes:

1. Evidence for •¹ may not appear until part (b).

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•² use result of part (a)	•² $\log_4 x + \log_4(x-6) = 2$	**5**
			•³ use laws of logarithms	•³ $\log_4 x(x-6) = 2$	
			•⁴ use laws of logarithms	•⁴ $x(x-6) = 4^2$	
			•⁵ write in standard quadratic form	•⁵ $x^2 - 6x - 16 = 0$	
			•⁶ solve for x and identify appropriate solution	•⁶ 8	

Notes:

2. •³& •⁴ can only be awarded for use of laws of logarithms applied to algebraic expressions of equivalent difficulty.
3. •⁴ is not available for $x(x-6) = 2^4$; however candidates may still gain •⁵& •⁶.
4. •⁶ is only available for solving a polynomial of degree 2 or higher.
5. •⁶ is not available for responses which retain invalid solutions.

Commonly Observed Responses:

Candidate A		**Candidate B**		**Candidate C**	
$\log_5 25 = 5$	•¹ ✗	$\log_5 25 = 2$	•¹ ✓	$\log_5 25 = 2$	•¹ ✓
$\log_4 x(x-6) = 5$	•² ✓1	$\log_4 x(x-6) = 2$	•² ✓	$\log_4 x(x-6) = 2$	•² ✓
	•³ ✓1		•³ ✓		•³ ✓
$x(x-6) = 4^5$	•⁴ ✓1	$x(x-6) = 8$	•⁴ ✗	$x(x-6) = 8$	•⁴ ✗
$x^2 - 6x - 1024 = 0$	•⁵ ✓1	$x^2 - 6x - 8 = 0$	•⁵ ✓1	$x^2 - 6x + 8 = 0$	•⁵ ✗
$35.14...$	•⁶ ✓1	$7.12...$	•⁶ ✓1	$x = 2, 4$	•⁶ ✗
				or	
				$x = \cancel{2}, \cancel{4}$	•⁶ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.	(a)		$\bullet^1$ value of a	$\bullet^1$ $a=4$	**3**
			$\bullet^2$ value of b	$\bullet^2$ $b=-5$	
			$\bullet^3$ calculate k	$\bullet^3$ $k=-\frac{1}{12}$	

Notes:

1. Evidence for the values of a and b may first appear in an expression for $f(x)$. Where marks have been awarded for a and b in an expression for $f(x)$ ignore any values attributed to a and b in subsequent working.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
Both roots interchanged				Using $(1,9)$	
$a=-5$	$\bullet^1$ ✗	$a=4$	$\bullet^1$ ✓	$a=-4$	$\bullet^1$ ✗
$b=4$	$\bullet^2$ $\boxed{\checkmark 1}$	$b=5$	$\bullet^2$ ✗	$b=5$	$\bullet^2$ $\boxed{\checkmark 1}$
$k=\frac{1}{6}$	$\bullet^3$ $\boxed{\checkmark 1}$	$k=-\frac{3}{16}$	$\bullet^3$ $\boxed{\checkmark 1}$	$k=\frac{9}{80}$	$\bullet^3$ $\boxed{\checkmark 1}$

Candidate D - BEWARE

Using $(0,9)$

$a=-5$	$\bullet^1$ ✗
$b=4$	$\bullet^2$ $\boxed{\checkmark 1}$
$k=\frac{9}{80}$	$\bullet^3$ ✗

Summary for expressions of $f(x)$ for $\bullet^1$ and $\bullet^2$:

signs correct, brackets correct

$f(x)=(x-4)(x+5)^2$ $\bullet^1$ ✓ $\bullet^2$ ✓

signs incorrect, brackets correct

$f(x)=(x+4)(x-5)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$

signs correct, brackets incorrect

$f(x)=(x+5)(x-4)^2$ $\bullet^1$ ✗ $\bullet^2$ $\boxed{\checkmark 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ state range of values	$\bullet^1$ $d>9$	**1**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

H

National
Qualifications
2016

2016 Mathematics

Higher Paper 2

Finalised Marking Instructions

SQA ©

Specific Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.	(a)	i	$\bullet^1$ state the midpoint M	$\bullet^1$ $(2,4)$	**1**
		ii	$\bullet^2$ calculate gradient of median $\bullet^3$ determine equation of median	$\bullet^2$ 4 $\bullet^3$ $y=4x-4$	**2**

Notes:

1. $\bullet^3$ is not available as a consequence of using a perpendicular gradient.
2. Accept any rearrangement of $y=4x-4$ for $\bullet^3$.
3. On this occasion, accept $y-4=4(x-2)$ or $y-(-4)=4(x-0)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.
4. $\bullet^3$ is only available as a consequence of using points M and P, or any other point which lies on PM, for example the midpoint $(1,0)$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^1$ calculate gradient of PR $\bullet^2$ use property of perpendicular lines $\bullet^3$ determine equation of line	$\bullet^1$ 1 $\bullet^2$ -1 stated or implied by $\bullet^6$ $\bullet^3$ $y=-x+6$	**3**

Notes:

5. $\bullet^6$ is only available as a consequence of using M and a perpendicular gradient.
6. Candidates who use a gradient perpendicular to QR cannot gain $\bullet^4$ but $\bullet^5$ and $\bullet^6$ are still available. See Candidate A.
7. Beware of candidates who use the coordinates of P and Q to arrive at $m=-1$. See Candidate B.
8. On this occasion, accept $y-4=-1(x-2)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Question			Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:					
Candidate A $m_{QR} = \frac{1}{4}$ •⁴ ✗ $m_{perp} = -4$ •⁵ $\boxed{\checkmark 1}$ $y = -4x + 12$ •⁶ $\boxed{\checkmark 1}$				**Candidate B - BEWARE** $m_{PQ} = \frac{2-(-4)}{-6-0}$ •⁴ ^ $= -1$ •⁵ ✗ $y - 4 = -1(x-2)$ •⁶ $\boxed{\checkmark 2}$ $y = -x + 6$ Note: •⁴ •⁵ and •⁶ may still be available for any candidate that demonstrates that PQ is also perpendicular to PR.	
	(c)			**Method 1**	3
			•¹ find the midpoint of PR	•¹ $(5,1)$	
			•² substitute x-coordinate into equation of L.	•² $y = -5 + 6 \ \ (1 = -x + 6)$	
			•³ verify y-coordinate and communicate conclusion	•³ $y = 1 \ (x = 5) \ \therefore$ L passes through the midpoint of PR	
				Method 2	
			•⁷ find the midpoint of PR	•⁷ $x + y = 6$ $\text{sub}\,(5,1)$	
			•⁸ substitute x and y coordinates into the equation of L	•⁸ $5 + 1 = 6$	
			•⁹ verify the point satisfies the equation and communicate conclusion	•⁹ $\therefore$ point $(5,1)$ satisfies equation.	
				Method 3	
			•⁷ find the midpoint of PR	•⁷ $(5,1)$	
			•⁸ find equation of PR	•⁸ $y = x - 4$	
			•⁹ use simultaneous equations and communicate conclusion	•⁹ $y = 1, \ x = 5 \ \therefore$ L passes through the midpoint of PR	

Question			Generic Scheme	Illustrative Scheme	Max Mark
				Method 4	
			•[7] find the midpoint of PR	•[7] $(5,1)$	
			•[8] find equation of perpendicular bisector of PR	•[8] $y-1=-1(x-5)\rightarrow y=-x+6$	
			•[9] communicate conclusion	•[9] The equation of the perpendicular bisector is the same as L therefore L passes through the midpoint of PR.	

Notes:

9. A relevant statement is required for •[9] to be awarded.
10. Erroneous working accompanied by a statement such as "L does not pass through the midpoint." does NOT gain •[9].
11. Beware of candidates substituting $(1,5)$ instead of $(5,1)$
12. On this occasion, for Method 3, at •[8] accept $y-1=1(x-5)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Commonly Observed Responses:

Candidate C

$(5,1)$ mid-point •[7] ✓

$y+x=6$

$\text{Sub}(5,1)$ •[8] ×

$5+1=6$

$\therefore$ point $(5,1)$ satisfies equation. •[9] ×

Candidate has substituted 5 for y and 1 for x.

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			•[1] use the discriminant •[2] simplify and apply the condition for no real roots •[3] state range	•[1] $(-2)^2-4(1)(3-p)$ •[2] $-8+4p<0$ •[3] $p<2$	**3**

Notes:

1. At the •[1] stage, treat $(-2)^2-4(1)3-p$ and $-2^2-4(1)(3-p)$ as bad form only if the candidate deals with the 'bad form' **term** correctly in the inequation at •[2].
2. If candidates have the condition 'discriminant = 0', then •[2] and •[3] are unavailable.
3. If candidates have the condition 'discriminant $>$ 0', 'discriminant $\geq$ 0' or 'discriminant $\leq$ 0' then •[2] is lost, but •[3] is available **provided the discriminant has been simplified correctly at** •[2].
4. If a candidate works with an equation, then •[2] and •[3] are unavailable. However, see Candidate D.

Commonly Observed Responses:

Candidate A

$(-2)^2-4(1)3-p$	•[1]	✓
$-8+4p<0$	•[2]	✓
$p<2$	•[3]	✓

Candidate B

$(-2)^2-4(1)(3-p)$	•[1]	✓
$-8-4p<0$	•[2]	✗
$p>-2$	•[3]	$\boxed{✓1}$

Candidate C

$(-2)^2-4(1)3-p$	•[1]	✗	
$-8-p<0$	•[2]	$\boxed{✓2}$	eased
$p>-8$	•[3]	$\boxed{✓2}$	eased

Candidate D - Special Case

$b^2-4ac<0$		
$(-2)^2-4(1)(3-p)=0$	•[1]	✓
$-8+4p=0$		
	•[2]	✓
$p=2$		
$p<2$	•[3]	$\boxed{✓2}$

•2 is awarded since the condition (first line), its application (final line) and the simplification of the discriminant all appear.

Candidate E

$-2^2-4(1)(3-p)$	•[1]	✓
$-8+4p<0$	•[2]	✓
$p<2$	•[3]	✓

Candidate F

$-2^2-4(1)(3-p)$	•[1]	✗
$-16+4p<0$	•[2]	$\boxed{✓2}$
$p<4$	•[3]	$\boxed{✓1}$

Candidate G

$-2^2-4(1)(3-p)=0$	•[1]	✓
$-8+4p=0$	•[2]	✗
$p=2$	•[3]	✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)	i	$\bullet^1$ know to substitute $x=-1$ $\bullet^2$ complete evaluation, interpret result and state conclusion	**Method 1** $\bullet^1$ $2(-1)^3-9\times(-1)^2+3\times(-1)+14$ $\bullet^2$ $=0$ $\therefore$ $(x+1)$ is a factor	**2**
			$\bullet^1$ know to use $x=-1$ in synthetic division $\bullet^2$ complete division, interpret result and state conclusion	**Method 2** $\bullet^1$ $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & & \\ \hline & 2 & -11 & & \end{array}$ $\bullet^2$ $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & 0 \end{array}$ remainder = 0 $\therefore$ $(x+1)$ is a factor	
			$\bullet^1$ start long division and find leading term in quotient $\bullet^2$ complete division, interpret result and state conclusion	**Method 3** $\bullet^1$ $\begin{array}{rl} & 2x^2 \\ (x+1) & \overline{\big)\,2x^3-9x^2+3x+14} \end{array}$ $\bullet^2$ $\begin{array}{rr} & 2x^2-11x+14 \\ (x+1) & \overline{\big)\,2x^3-9x^2+3x+14} \\ & \underline{2x^3+2x^2} \\ & -11x^2+3x \\ & \underline{-11x^2-11x} \\ & 14x+14 \\ & \underline{14x+14} \\ & 0 \end{array}$ remainder = 0 $\therefore$ $(x+1)$ is a factor	

Question			Generic Scheme	Illustrative Scheme	Max Mark
Notes:					
1. Communication at $\bullet^2$ must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded. 2. Accept any of the following for $\bullet^2$: • '$f(-1)=0$ so $(x+1)$ is a factor' • 'since remainder = 0, it is a factor' • the 0 from any method linked to the word 'factor' by eg 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$' 3. Do not accept any of the following for $\bullet^2$: • double underlining the zero or boxing the zero without comment • '$x=1$ is a factor', '$(x-1)$ is a factor', '$(x-1)$ is a root', '$x=-1$ is a root', '$(x+1)$ is a root' • the word 'factor' only with no link					
Commonly Observed Responses:					
		ii	$\bullet^3$ state quadratic factor $\bullet^4$ find remaining linear factors or substitute into quadratic formula $\bullet^5$ state solution	$\bullet^3$ $2x^2-11x+14$ $\bullet^4$ $....(2x-7)(x-2)$ or $\dfrac{11\pm\sqrt{(-11)^2-4\times2\times14}}{2\times2}$ $\bullet^5$ $x=-1,\ 2,\ 3{\cdot}5$	**3**
Notes:					
4. On this occasion, the appearance of "$=0$" is not required for $\bullet^5$ to be awarded. 5. Be aware that the solution, $x=-1,\ 2,\ 3{\cdot}5$, may not appear until part (b).					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)	(i)	•[6] state coordinates	•[6] $(-1,0)$ and $(2,0)$	**1**

Notes:

6. '-1 and 2' does not gain •[6]
7. $x=-1, y=0$ **and** $x=2, y=0$ gains •[6]

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
		(ii)	•[7] know to integrate with respect to x	•[7] $\int\left(2x^3-9x^2+3x+14\right)dx$	**4**
			•[8] integrate	•[8] $\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$	
			•[9] interpret limits and substitute	•[9] $\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)-\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)$	
			•[10] evaluate integral	•[10] 27	

Candidate A

$\int\left(2x^3-9x^2+3x+14\right)dx$ •[7] ✓

$\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓

27 •[9] ^ •[10] $\boxed{✓1}$

Candidate B

$\int\left(2x^3-9x^2+3x+14\right)dx$ •[7] ✓

$\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓

$\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)-\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)$ •[9] ✗

-27, hence area is 27 •[10] $\boxed{✓1}$

However, $-27=27$ does not gain •[10].

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Candidate C $\int ... = -27$ cannot be negative so $= 27$ $\bullet^{10}$ ✗ However, $\int ... = -27$ so Area $= 27$ $\bullet^{10}$ ✓		

Notes:

8. $\bullet^7$ is not available to candidates who omit ‘ dx ‘
9. Do not penalise the absence of brackets at the $\bullet^7$ stage
10. Where a candidate differentiates one or more terms at $\bullet^8$, then $\bullet^8$, $\bullet^9$ and $\bullet^{10}$ are not available.
11. Candidates who substitute limits without integrating do not gain $\bullet^8$, $\bullet^9$ or $\bullet^{10}$.
12. For candidates who make an error in (a), $\bullet^9$ is available only if the lower limit is negative and the upper limit is positive.
13. Do not penalise the inclusion of ‘$+c$’.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		●[1] centre of C_1	●[1] $(-5,6)$	**4**
			●[2] radius of C_1	●[2] 3	
			●[3] centre of C_2	●[3] $(3,0)$	
			●[4] radius of C_2	●[4] 5	

Notes:

Commonly Observed Responses:

	(b)		●[5] calculate the distance between the centres	●[1] 10	**3**
			●[6] calculate the sum of the radii	●[2] 8	
			●[7] interpret significance of calculations	●[3] $8<10 \therefore$ the circles do not intersect	

Notes:

1. For ●[7] to be awarded a comparison must appear.
2. Candidates who write '$r_1+r_2<\mathrm{D}$', or similar, must have identified the value of r_1+r_2 and the value of D somewhere in their solution for ●[7] to be awarded.
3. Where earlier errors lead to the candidate dealing with non-integer values, do not penalise inaccuracies in rounding unless they lead to an inconsistent conclusion.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.	(a)		•¹ find $\overrightarrow{AB}$	•¹ $\begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$	2
			•² find $\overrightarrow{AC}$	•² $\begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$	
Notes:					
1. For candidates who find **both** $\overrightarrow{BA}$ **and** $\overrightarrow{CA}$ correctly, only •² is available (repeated error). 2. Accept vectors written horizontally.					
Commonly Observed Responses:					
	(b)		**Method 1**	**Method 1**	4
			•¹ evaluate $\overrightarrow{AB}.\overrightarrow{AC}$	•¹ $\overrightarrow{AB}.\overrightarrow{AC} = 16 - 128 + 32 = -80$	
			•² evaluate $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{AC}\rvert$	•² $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = 18$	
			•³ use scalar product	•³ $\cos BAC = \dfrac{-80}{18 \times 18}$	
			•⁴ calculate angle	•⁴ $104 \cdot 3°$ or $1 \cdot 82$ radians	
			Method 2	**Method 2**	
			•³ calculate length of BC	•³ $BC = \sqrt{808}$	
			•⁴ calculate lengths of AB and AC	•⁴ $AB = AC = 18$	
			•⁵ use cosine rule	•⁵ $\cos BAC = \dfrac{18^2 + 18^2 - \sqrt{808}^2}{2 \times 18 \times 18}$	
			•⁶ calculate angle	•⁶ $104 \cdot 3°$ or $1 \cdot 82$ radians	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

3. Accept $\sqrt{324}$ at $\bullet^4$ and $\bullet^5$.
4. $\bullet^5$ is not available to candidates who simply state the formula $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$.

   However $\cos\theta = \dfrac{-80}{18\times 18}$ is acceptable. Similarly for Method 2.
5. Accept correct answers rounded to $104°$ or $1\cdot 8$ radians.
6. Due to $\overrightarrow{AB}$ and $\overrightarrow{AC}$ having equal magnitude, $\bullet^4$ is not available unless both $|\overrightarrow{AB}|$ **and** $|\overrightarrow{AC}|$ have been stated.
7. $\bullet^6$ is only available as a result of using a valid strategy.
8. $\bullet^6$ is only available for a single angle.
9. For a correct answer with no working award 0/4.

Commonly Observed Responses:

Candidate A

$\overrightarrow{BA}.\overrightarrow{AC} = -16+128-32 = 80$	$\bullet^3$ ✗
$\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = 18$	$\bullet^4$ $\boxed{✓1}$
$\cos\theta = \dfrac{80}{18\times 18}$	$\bullet^5$ $\boxed{✓1}$
	$\bullet^6$ ✗
$75\cdot 7$ or $1\cdot 32$ radians	

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)		●¹ state the number	●¹ 200	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●² interpret context and form equation ●³ knowing to use logarithms appropriately. ●⁴ simplify ●⁵ evaluate t	●² $2 = e^{0\cdot107t}$ ●³ $\ln 2 = \ln\left(e^{0\cdot107t}\right)$ ●⁴ $\ln2 = 0\cdot107t$ ●⁵ $t = 6\cdot478...$	**4**

Notes:

1. Accept $400 = 200e^{0\cdot107t}$ or equivalent for ●²
2. Any base may be used at the ●³ stage.
3. ●³ may be assumed by ●⁴.
4. Accept $t = 6\cdot5$.
5. At ●⁵ ignore incorrect units. However, see Candidates B and C.
6. The calculation at ●⁵ must involve the evaluation of a logarithm within a valid strategy for ●⁵ to be awarded.
7. Candidates who take an iterative approach to arrive at $t = 6\cdot5$ gain ●² only. However, if, in the iterations, $B(t)$ is evaluated for $t = 6\cdot45$ and $t = 6\cdot55$ then award 4/4.

Commonly Observed Responses:

Candidate A

$2 = e^{0\cdot107t}$ ●² ✓

$\log_{10} 2 = \log_{10}\left(e^{0\cdot107t}\right)$ ●³ ✓

$\log_{10} 2 = 0\cdot107t\log_{10}e$ ●⁴ ✓

$t = 6\cdot478\ldots$ ●⁵ ✓

Candidate B

$t = 6\cdot48$ hours ●⁵ ✓

$t = 6$ hours 48 minutes

Candidate C

$\ln(2) = 0\cdot107t$ ●⁴ ✓

$t = 6$ hours 48 minutes ●⁵ ✗

Candidate D

$400 = 200e^{0\cdot107t}$ ●² ✓

$e^{0\cdot107t} = 2$ ●³ ^

$t = 6\cdot48$ hours $\boxed{✓1}$ ●⁴ $\boxed{✓1}$ ●⁵

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		$\bullet^1$ expression for length in terms of x and y $\bullet^2$ obtain an expression for y $\bullet^3$ demonstrate result	$\bullet^1$ $9x+8y$ $\bullet^2$ $y=\dfrac{108}{6x}$ $\bullet^3$ $\mathrm{L}(x)=9x+8\left(\dfrac{108}{6x}\right)$ leading to $\mathrm{L}(x)=9x+\dfrac{144}{x}$	**3**

Notes:

1. The substitution for y at $\bullet^3$ must be clearly shown for $\bullet^3$ to be available.
2. For candidates who omit some, or all, of the internal fencing, only $\bullet^2$ is available.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^4$ know to and start to differentiate $\bullet^5$ complete differentiation $\bullet^6$ set derivative equal to 0 $\bullet^7$ obtain for x $\bullet^8$ verify nature of stationary point $\bullet^9$ interpret and communicate result	$\bullet^4$ $\mathrm{L}'(x)=9\ldots$ $\bullet^5$ $L'(x)=9-\dfrac{144}{x^2}$ $\bullet^6$ $9-\dfrac{144}{x^2}=0$ $\bullet^7$ $x=4$ $\bullet^8$ Table of signs for a derivative – see the additional page. $\bullet^9$ Minimum at $x=4$ **or** $\bullet^8$ $\mathrm{L}''(x)=\dfrac{288}{x^3}$ $\bullet^9$ $\mathrm{L}''(4)>0$ $\therefore$ minimum Do not accept $\dfrac{d^2y}{dx^2}=\ldots$	**6**

Question	Generic Scheme	Illustrative Scheme	Max Mark
Notes:			
3. For candidates who integrate at the $\bullet^4$ stage $\bullet^5$, $\bullet^6$, $\bullet^7$, $\bullet^8$ and $\bullet^9$ are unavailable. 4. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are only available for working with a derivative which contains a term with an index ≤ -2. 5. At $\bullet^5$ and $\bullet^6$ accept $-144x^{-2}$. 6. $\sqrt{\frac{144}{9}}$ must be simplified at the $\bullet^7$, $\bullet^8$ or $\bullet^9$ stage for $\bullet^7$ to be awarded. 7. $\bullet^9$ is not available to candidates who consider a value ≤ 0 in the neighbourhood of 4. 8. A candidate's table of signs must be valid and legitimately lead to a minimum for $\bullet^9$ to be awarded. 9. $\bullet^9$ is not available to candidates who state the minimum exists at a negative value of x.			
Commonly Observed Responses:			

Table of signs for a derivative

Accept

x	-4^-	-4	-4^+
$L'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	4^-	4	4^+
$L'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Here, for exemplification, tables of signs considering both roots separately have been displayed. However, in this question, it was only expected that candidates would consider the positive root for $\bullet^8$. Do not penalise the consideration of the negative root.

x	$\rightarrow$	-4	$\rightarrow$
$L'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	4	$\rightarrow$
$L'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Arrows are taken to mean "in the neighbourhood of"

x	a	-4	b	c	4	d
$L'(x)$	$+$	0	$-$	$-$	0	$+$
Shape or Slope	/	—	\	\	—	/

Where: $a < -4$, $-4 < b < 0$, $0 < c < 4$, $d > 4$

Do not Accept

x	a	-4	b	4	c
$L'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-4 < b < 4$' is not acceptable.

x	$\rightarrow$	-4	$\rightarrow$	4	$\rightarrow$
$L'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-4 \rightarrow 4$' is not acceptable.

General Comments

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $L'(x)$ in the table is an acceptable alternative to writing '+' or '-' signs.
- Acceptable alternatives for $L'(x)$ are: L' , $\frac{dL}{dx}$ or $9-\frac{144}{x^2}$. **DO NOT** accept $\frac{dy}{dx}$ or $f'(x)$.

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•¹ use compound angle formula	•¹ $k\cos x\cos a - k\sin x\sin a$ **stated explicitly**	**4**
			•² compare coefficients	•² $k\cos a = 5, k\sin a = 2$ **stated explicitly**	
			•³ process for k	•³ $k = \sqrt{29}$	
			•⁴ process for a and express in required form	•⁴ $\sqrt{29}\cos(x + 0\cdot38)$	

Notes:

1. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the •² stage both contain k.
2. $\sqrt{29}\cos x\cos a - \sqrt{29}\sin x\sin a$ or $\sqrt{29}(\cos x\cos a - \sin x\sin a)$ is acceptable for •¹ and •³.
3. Accept $k\cos a = 5, -k\sin a = -2$ for •².
4. •² is not available for $k\cos x = 5, k\sin x = 2$, however, •⁴ is still available.
5. •³ is only available for a single value of $k, k > 0$.
6. Candidates who work in degrees and do not convert to radian measure do not gain •⁴.
7. Candidates may use any form of the wave equation for •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted in the form $k\cos(x + a)$.
8. Accept any answer for a that rounds to $0\cdot38$.
9. Evidence for •⁴ may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A	**Candidate B**
•¹ ^	$k\cos x\cos a - k\sin x\sin a$ •¹ ✓
$\sqrt{29}\cos a = 5$ •² ✓	$\cos a = 5$
$\sqrt{29}\sin a = 2$ •³ ✓	$\sin a = 2$ •² ✗
$\tan a = \frac{2}{5}$, $a = 0\cdot3805...$	•³ ✓
$\sqrt{29}\cos(x + 0\cdot38)$ •⁴ ✓	$\tan a = \frac{2}{5}$
	$a = 0\cdot38$ — Not consistent with equations at •².
	$\sqrt{29}\cos(x + 0\cdot38)$ •⁴ ✗

Responses with the correct expansion of $k\cos(x + a)$ but errors for either •² or •⁴.

Candidate C	**Candidate D**	**Candidate E**
$k\cos a = 5, k\sin a = 2$ •² ✓	$k\cos a = 2, k\sin a = 5$ •² ✗	$k\cos a = 5, k\sin a = -2$ •² ✗
$\tan a = \frac{5}{2}$ •⁴ ✗	$\tan a = \frac{5}{2}$, $a = 1\cdot19$	$\tan a = \frac{-2}{5}$
$a = 1\cdot19$	$\sqrt{29}\cos(x + 1\cdot19)$ •⁴ $\boxed{✓1}$	$\sqrt{29}\cos(x + 5\cdot90)$ •⁴ $\boxed{✓1}$

Question	Generic Scheme	Illustrative Scheme	Max Mark

Responses with the incorrect labelling; $k\cos A\cos B - k\sin A\sin B$ from formula list.

Candidate F	Candidate G	Candidate H
$k\cos A\cos B - k\sin A\sin B$ ●^{1} ✗	$k\cos A\cos B - k\sin A\sin B$ ●^{1} ✗	$k\cos A\cos B - k\sin A\sin B$ ●^{1} ✗
$k\cos a = 5,\ k\sin a = 2$ ●^{2} ✓	$k\cos x = 5,\ k\sin x = 2$ ●^{2} ✗	$k\cos B = 5,\ k\sin B = 2$ ●^{2} $\boxed{\checkmark 1}$
$\tan a = \frac{2}{5},\ a = 0\cdot3805...$	$\tan x = \frac{2}{5},\ x = 0\cdot3805...$	$\tan B = \frac{2}{5},\ B = 0\cdot3805...$
$\sqrt{29}\cos\left(x + 0\cdot38\right)$ ●^{3} ✓ ●^{4} ✓	$\sqrt{29}\cos\left(x + 0\cdot38\right)$ ●^{3} ✓ ●^{4} $\boxed{\checkmark 1}$	$\sqrt{29}\cos\left(x + 0\cdot38\right)$ ●^{3} ✓ ●^{4} $\boxed{\checkmark 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●^{5} equate to 12 and simplify constant terms	●^{5} $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	**4**
			●^{6} use result of part (a) and rearrange	●^{6} $\cos\left(x + 0\cdot3805\ldots\right) = \frac{2}{\sqrt{29}}$	
			●^{7} solve for $x + a$	●^{7} $1\cdot1902\ldots$, ●^{8} $5\cdot0928\ldots$	
			●^{8} solve for x	●^{8} $0\cdot8097\ldots$, $4\cdot712\ldots$	

Notes:

10. The values of x may be given in radians or degrees.
11. Do not penalise candidates who attribute the values of x to the wrong points.
12. Accept any answers, in degrees or radians, that round correct to one decimal place.
13. ●^{4} is unavailable for candidates who give their answer in degrees in part (a) and in part (b). ●^{4} is unavailable for candidates who give their answer in degrees in part (a) and radians in part (b). ●^{8} is unavailable for candidates who give their answer in radians in part (a) and degrees in part (b).

Conversion Table:

Degrees	Radians
$21\cdot8$	$0\cdot3805...$
$46\cdot4$	$0\cdot8097...$
$68\cdot2$	$1\cdot190...$
270	$4\cdot712...$or $\frac{3\pi}{2}$
$291\cdot8$	$5\cdot0928...$

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			•¹ write in integrable form	•¹ $2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$	4
			•² integrate one term	•² $\frac{4}{3}x^{\frac{3}{2}}$ or $2x^{\frac{1}{2}}$	
			•³ complete integration	•³ $2x^{\frac{1}{2}} + c$ or $\frac{4}{3}x^{\frac{3}{2}} + c$	
			•⁴ state expression for $f(x)$	•⁴ $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2$	

Notes:

1. For candidates who do not attempt to write $f'(x)$ as the sum of two integrable terms, award 0/4.
2. •² and •³ are only available for integrating terms involving fractional indices.
3. The term integrated at •³ must have an index of opposite sign to that of the term integrated at •².
4. For candidates who differentiate one term, only •¹ and •² are available.
5. For candidates who differentiate both terms, only •¹ is available.
6. For •⁴ . accept ‘ $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c,\ c = -2$ ’
7. Candidates must simplify coefficients in **<u>their</u>** final line of working for the last mark available for that line of working to be awarded.

Commonly Observed Responses:

Candidate A		Candidate B	
$f'(x) = 2x + x^{-\frac{1}{2}}$	•¹ ✗	$f'(x) = 2x + x^{-\frac{1}{2}}$	•¹ ✗
$x^2 + 2x^{\frac{1}{2}} + c$	•² ✓ •³ $\boxed{✓2}$	$x^2 + 2x^{\frac{1}{2}}$	•² ✓ •³ ✗
$f(x) = x^2 + 2x^{\frac{1}{2}} - 47$	•⁴ $\boxed{✓1}$	$f(x) = x^2 + 2x^{\frac{1}{2}}$	•⁴ ^
Candidate C		**Candidate D**	
$f'(x) = 2x^{\frac{1}{2}} + 1$	•¹ ✗	$f'(x) = \dfrac{2x+1}{x^{\frac{1}{2}}}$	•¹ ^
$\frac{4}{3}x^{\frac{3}{2}} + x + c$	•² ✓ •³ $\boxed{✓2}$	$\dfrac{x^2 + x}{2x^{\frac{3}{2}}} + c$	•² ✗ •³ ✗ •⁴ ✗ See Note 1
$f(x) = \frac{4}{3}x^{\frac{3}{2}} + x - 5$	•⁴ $\boxed{✓1}$	$f(x) = \dfrac{x^2 + x}{2x^{\frac{3}{2}}} + \dfrac{115}{3}$	

Question			Generic Scheme	Illustrative Scheme	Max Mark

Candidate E

$f'(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ $\bullet^1$ ✓

$= \dfrac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$ $\bullet^2$ ✓

$\bullet^3$ $\boxed{✓2}$ $\bullet^4$ ^

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		$\bullet^1$ Start to differentiate $\bullet^2$ Complete differentiation	$\bullet^1$ $\frac{1}{2}(x^2+7)^{-\frac{1}{2}}$..... $\bullet^2$ ....$\times\ 2x$	**2**

Notes:

1. On this occasion there is no requirement to simplify coefficients.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^3$ link to (a) and integrate	$\bullet^3$ $4(x^2+7)^{\frac{1}{2}}\ (+\ c)$	**1**

Notes:

2. A candidate's answer at $\bullet^3$ must be consistent with earlier working.

Commonly Observed Responses:

Candidate A

$\int 4x(x^2+7)^{\frac{-1}{2}}\,dx$

$= \dfrac{4x(x^2+7)^{\frac{1}{2}}}{\frac{1}{2}\times 2x} + c$

$= \dfrac{4x(x^2+7)^{\frac{1}{2}}}{x} + c$

$= 4(x^2+7)^{\frac{1}{2}} + c$ $\bullet^3$ ×

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		•[1] substitute for $\sin 2x$ and $\tan x$	•[1] $(2\sin x\cos x)\times\dfrac{\sin x}{\cos x}$	**4**
			•[2] simplify	•[2] $2\sin^2 x$	
			•[3] use an appropriate substitution	•[3] $2\left(1-\cos^2 x\right)$ or $1-\left(1-2\sin^2 x\right)$	
			•[4] simplify and communicate result	•[4] $1-\cos 2x = 1-\cos 2x$ or $2\sin^2 x = 2\sin^2 x$ $\therefore$ Identity shown	

Notes:

1. •[1] is not available to candidates who simply quote $\sin 2x = 2\sin x\cos x$ and $\tan x = \dfrac{\sin x}{\cos x}$ without substituting into the identity.
2. •[4] is not available to candidates who work throughout with A in place of x.
3. •[3] is not available to candidates who simply quote $\cos 2x = 1-2\sin^2 x$ without substituting into the identity.
4. On this occasion, at •[4] do not penalise the omission of 'LHS = RHS' or a similar statement.

Commonly Observed Responses:

Candidate A		Candidate B	
$\sin 2x\tan x = 1-\cos 2x$		$\sin 2x\tan x = 1-\cos 2x$	
$2\sin x\cos x\times\dfrac{\sin x}{\cos x} = 1-\cos 2x$	•[1] ✓	$\sin 2x\tan x = 1-(1-2\sin^2 x)$	•[1] ^ •[2] ^
$2\sin^2 x = 1-\cos 2x$	•[2] ✓	$\sin 2x\tan x = 2\sin^2 x$	•[3] ✓
$2\sin^2 x - 1 = -\cos 2x$	•[3] ✗ •[4] ✗	$\tan x = \dfrac{2\sin^2 x}{2\sin x\cos x}$	•[4] ✗
$-(1-2\sin^2 x) = -\cos 2x$		$\tan x = \tan x$	
$-\cos 2x = -\cos 2x$			
In proving the identity, candidates must work with both sides independently. ie in each line of working the LHS must be equivalent to the left hand side of the line above.			

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●5 link to (a) and substitute ●6 differentiate	●5 $f(x)=1-\cos 2x$ or $f(x)=2\sin^2 x$ ●6 $f'(x)=2\sin 2x$ or $f'(x)=4\sin x\cos x$	**2**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]